Telecom Network Systems

Wavelength Dispersion

1) Phase velocity: $V_p = \frac{c}{n} = \frac{3 \cdot 10^8 \text{ m/s}}{1.4448} = 2.076 \cdot 10^8 \text{ m/s}$ ← see appendix A

Group velocity: $V_g = \frac{c}{n_0} = \frac{3 \cdot 10^8 \text{ m/s}}{1.4625} = 2.05 \; 10^8 \text{ m/s}$

see appendix A

2) Very difficult to see 2nd derivative from Figure 26. Better: http://refractiveindex.info

D of fused silica glass at 1.5 μm

$$D = 18.583 \; \frac{ps}{nm \cdot km} \qquad \text{(see appendix B)}$$

$1.8583 \; 10^{-5} \; s/m^2$

3)
$$\beta_2 \equiv \frac{d^2 n(\omega)}{d\omega^2} = \frac{d^2 n(\lambda)}{d\lambda^2} \cdot \frac{d^2\lambda}{d\omega^2}$$

$$= -\frac{c}{\lambda} \cdot \left(-\frac{\lambda}{c} \frac{d^2 n(\lambda)}{d\lambda^2}\right) \cdot \frac{d^2\lambda}{d\omega^2}$$

$$= -\frac{c}{\lambda} \frac{d^2\lambda(\omega)}{d\omega^2} \cdot D \qquad \text{(I)}$$

$$\omega = \frac{2\pi c}{\lambda} \Rightarrow \lambda = \frac{2\pi c}{\omega}, \quad \frac{d\lambda}{d\omega} = -\frac{2\pi c}{\omega^2}, \quad \frac{d^2\lambda}{d\omega^2} = \frac{2 \cdot 2\pi c}{\omega^3} =$$

$$\frac{1}{\omega^3} = \frac{\lambda^3}{(2\pi)^3 c^3}$$

$$= 2 \cdot 2\pi \cdot c \cdot \frac{\lambda^3}{(2\pi)^3 c^3} = \frac{2\lambda^3}{(2\pi)^2 c^2}$$

In (I)

$$\beta_2 = -\frac{c}{\lambda}\frac{2\lambda^3}{(2\pi)^2c^2}\cdot D = \frac{-\lambda^2}{2\pi^2 c}\cdot D$$

?

Second method

$$\beta \equiv n\frac{\omega}{c} \quad . \quad \beta_i \equiv \frac{d^i\beta}{d\omega^i}$$

$$\omega = \frac{2\pi c}{\lambda}, \quad \lambda = \frac{2\pi c}{\omega}, \quad \frac{d\lambda}{d\omega} = -\frac{\lambda^2}{2\pi c}$$

$$\beta_1 = \frac{d\beta}{d\omega} = \left(\frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}\right) \qquad (I)$$

$$\frac{dn}{d\omega} = \frac{dn}{d\lambda}\cdot\frac{d\lambda}{d\omega} = \left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{dn}{d\lambda} \quad \text{in } I$$

$$\beta_1 = \left(\frac{n}{c} + \frac{2\pi c}{\lambda c}\cdot\left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{dn}{d\lambda}\right)$$

$$= \left(\frac{n}{c} - \frac{\lambda}{c}\frac{dn}{d\lambda}\right)$$

$$\beta_2 \equiv \frac{d\beta_1}{d\omega} = \frac{d\beta_1}{d\lambda}\cdot\frac{d\lambda}{d\omega} = \left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{d\beta_1}{d\lambda}$$

$$= \left(-\frac{\lambda^2}{2\pi c}\right)\cdot\frac{d}{d\lambda}\left(\frac{n}{c} - \frac{\lambda}{c}\frac{dn}{d\lambda}\right)$$

$$= \left(-\frac{\lambda^2}{2\pi c}\right)\frac{1}{c}\left(\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2}\right)$$

$$= \frac{\lambda^3}{2\pi c^2}\cdot\frac{d^2n}{d\lambda^2} = -\frac{\lambda^2}{2\pi c}\cdot\left(-\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}\right) = -\frac{\lambda^2}{2\pi c}\cdot D$$

4) $T_0 = 7.5 \text{ ps} = 7.5 \cdot 10^{-12} s$

$$D = 18.583 \frac{ps}{nm\ km} = 18.583 \frac{10^{-12}}{10^{-9} \cdot 10^{3}} \frac{s}{m^2}$$

$$= 18.583\ 10^{-6}\ s/m^2$$

$$\beta_2 = -\frac{\lambda^2}{2\pi c} D = -\frac{(1.5\ 10^{-6}\ m)^2}{2\pi \cdot 3 \cdot 10^{8}\ m/s} \cdot 18.583\ 10^{-6}\ s/m^2$$

$$= -2.218 \cdot 10^{-26}\ s^2/m$$

$$L_D = \frac{T_0^2}{\beta_2} = \frac{(7.5 \cdot 10^{-12})^2 s^2}{2.218\ 10^{-26}\ s^2/m} = 2.536\ km$$

5) $T(z) = T_0 \sqrt{1 + \left(\frac{z}{L_D}\right)^2} = 20 \text{ ps}$

$$z = L_D \sqrt{\left(\frac{T(z)}{T_0}\right)^2 - 1} = 2536 \cdot 10^{3} \sqrt{\left(\frac{20\ ps}{7.5\ ps}\right)^2 - 1}$$

$$= 6.27\ km$$

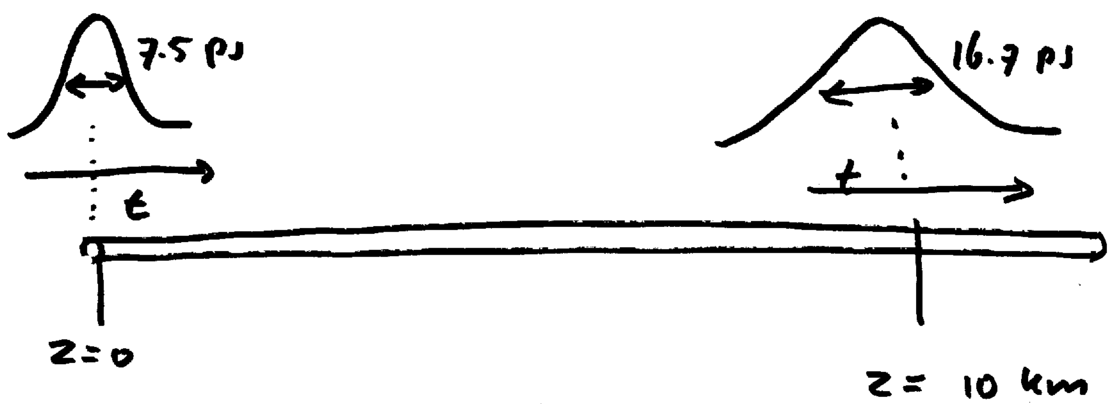
6) Because the waveguide is zero, the chromatic dispersion is equal to the material dispersion (of Fig. 2b). We can use the dispersion found in question 2

$$\left.\begin{aligned} D &= 18.583 \ \frac{ps}{nm \cdot km} \\ d &= 20 \ km \\ \Delta\lambda &= 20 \ pm \end{aligned}\right\} \quad \begin{aligned} C.D. &= D \cdot d \cdot \Delta\lambda \\ &= 7.43 \ ps \end{aligned}$$

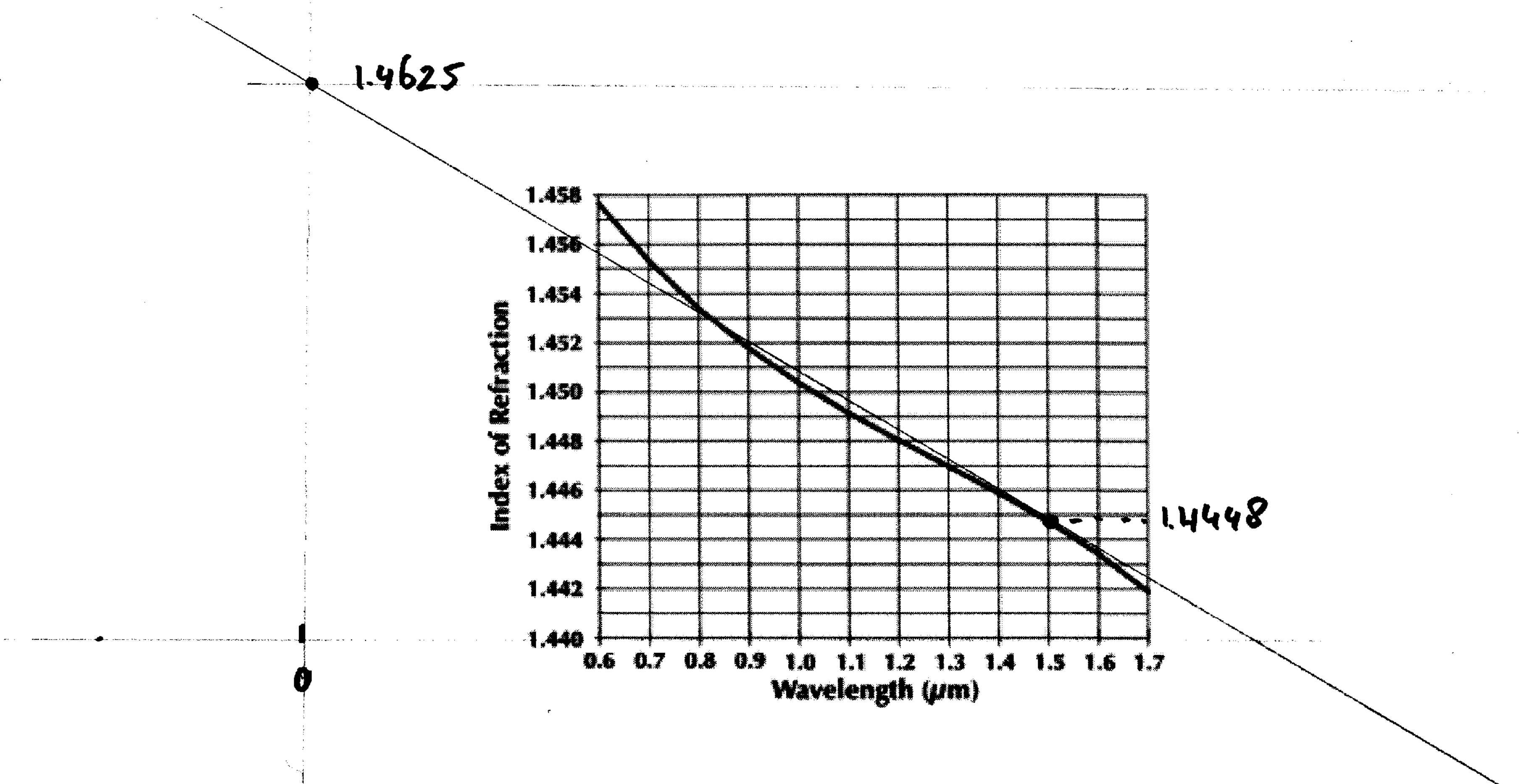
7)

$$\left.\begin{aligned} T_0 &= 7.5 \ ps \\ z &= 10 \ km \\ (\beta_2 &= 2.218 \cdot 10^{-26} \ s^2/m) \\ L_D &= 2.536 \cdot 10^{3} \ m \end{aligned}\right| \quad \text{from question 4}$$

$$T(z) = T_0 \cdot \sqrt{1 + \left(\frac{z}{L_D}\right)^2} = 16.7 \ ps$$



Appendix A



1.4625
1.4448
0
Index of Refraction
1.440
1.442
1.444
1.446
1.448
1.450
1.452
1.454
1.456
1.458
0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7
Wavelength (μm)

Appendix B

Optical constants of Fused silica (fused quartz)

Malitson 1965 - n 0.21-3.71 µm

Wavelength: 1.5 µm (0.21 – 3.71)

Refractive index

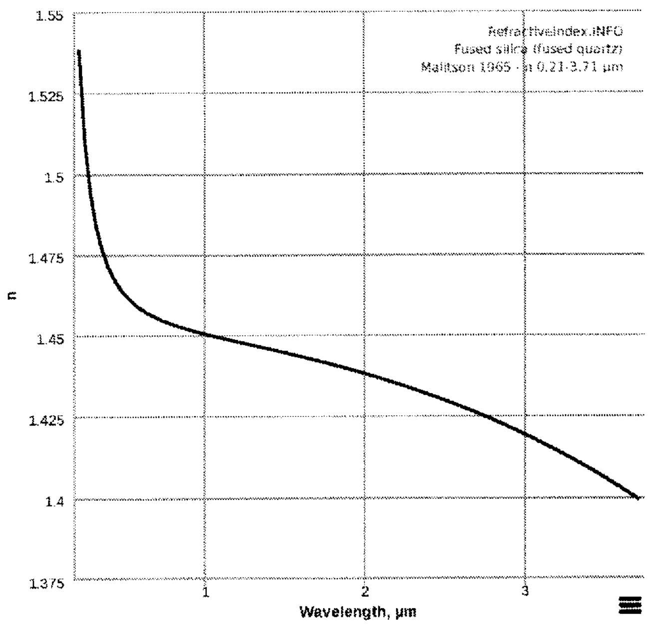
$n = 1.4446$

Other optical constants

Abbe number
$V_d = 67.82$

Chromatic dispersion
$dn/d\lambda = -0.011783\ \mu m^{-1}$

Group velocity dispersion
$GVD = -22.197\ fs^2/mm$
$D = 18.583$ ps/(nm km)



☐ log ☐ eV

Dispersion formula

$$n^2 - 1 = \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161^2}$$

Comments

Room temperature

References

I. H. Malitson. Interspecimen Comparison of the Refractive Index of Fused Silica, *J. Opt. Soc. Am.* **55**, 1205-1208 (1965)

Reflection calculator

Angle of incidence (0~90°): 0

Direction: ◉in ○out

Reflectance (at 1.5 µm)

$R = 0.033079$

Reflection phase

$\phi = 180°$

Brewster's angle

$\theta_B = 55.308°$